

Comments on atmospheric neutrino oscillation scenarios with large $\nu_\mu \leftrightarrow \nu_e$ transitions

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Abstract

The evidence for ν_μ disappearance in the Super-Kamiokande atmospheric neutrino experiment and the negative searches for ν_e disappearance in the CHOOZ reactor experiment can be easily reconciled by assuming oscillations with large amplitude in the $\nu_\mu \leftrightarrow \nu_\tau$ channel and small (or null) amplitude in the $\nu_\mu \leftrightarrow \nu_e$ channel. It has been claimed, however, that some oscillation scenarios with large $\nu_\mu \leftrightarrow \nu_e$ mixing can also be constructed in agreement with the present data. We investigate quantitatively two such scenarios: (a) threefold maximal mixing; and (b) attempts to fit all sources of evidence for oscillations (solar, atmospheric, and accelerator) with three neutrinos. By using mainly Super-Kamiokande data, we show that the case (a) is disfavored, and that the case (b) is definitely ruled out.

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I. INTRODUCTION

As well known, the evidence for atmospheric ν_μ disappearance observed in the Super-Kamiokande (SK) experiment [1] can be explained by two-family oscillations with large mixing in the $\nu_\mu \leftrightarrow \nu_\tau$ channel [2]. When this solution, known as twofold maximal mixing, is assumed, the relevant flavor oscillation probabilities $P_{\alpha\beta}$ read:

$$P_{ee} = 1 , \quad (1)$$

$$P_{\mu e} = 0 , \quad (2)$$

$$P_{\mu\mu} = 1 - \sin^2(\Delta m_{\text{atm}}^2 L/4E) , \quad (3)$$

where L and E are the neutrino pathlength and energy, respectively, and the experimentally inferred value of the neutrino squared mass difference is [3,4]

$$\Delta m_{\text{atm}}^2 \sim 3 \times 10^{-3} \text{ eV}^2 \quad (4)$$

within a factor of about two.

Oscillations in the $\nu_\mu \leftrightarrow \nu_\tau$ channel are not affected by matter effects, and are consistent with the negative results of the CHOOZ ν_e disappearance experiment [5,6], which requires $P_{ee} \sim 1$ for $\Delta m^2 \gtrsim 10^{-3} \text{ eV}^2$. Conversely, two-family oscillations in the $\nu_\mu \leftrightarrow \nu_e$ channel are heavily affected by matter effects, do not provide a good fit to SK data [3,4], and are independently excluded by CHOOZ [5].

Going beyond pure $\nu_\mu \leftrightarrow \nu_\tau$ oscillations, one should consider the possibility that the flavor eigenstates (ν_e, ν_μ, ν_τ) are linear combinations of three states (ν_1, ν_2, ν_3) with definite masses $m_1 \leq m_2 \leq m_3$ through a unitary matrix $U_{\alpha i}$:

$$U_{\alpha i} = \begin{pmatrix} c_\omega c_\phi & s_\omega c_\phi & s_\phi \\ -s_\omega c_\psi - c_\omega s_\psi s_\phi & c_\omega c_\psi - s_\omega s_\psi s_\phi & s_\psi c_\phi \\ s_\omega s_\psi - c_\omega c_\psi s_\phi & -c_\omega s_\psi - s_\omega c_\psi s_\phi & c_\psi c_\phi \end{pmatrix} \quad (5)$$

where (ω, ϕ, ψ) are the mixing angles in the standard parametrization, and a possible CP violating phase has been neglected. The parameter Δm_{atm}^2 should then be taken equal to one of the following squared mass differences:

$$\delta m^2 = m_2^2 - m_1^2 , \quad (6)$$

$$m^2 = m_3^2 - m_2^2 , \quad (7)$$

assuming that $\delta m^2 \ll m^2$.

Different choices for Δm_{atm}^2 have different implications for three-flavor scenarios. In particular, if Δm_{atm}^2 is taken within CHOOZ bounds (i.e., $\gtrsim 10^{-3} \text{ eV}^2$), then large ν_μ disappearance in SK can be generated only by dominant $\nu_\mu \leftrightarrow \nu_\tau$ oscillations, while $\nu_\mu \leftrightarrow \nu_e$ transitions must be small (or absent). This is the scenario favored by the quantitative analyses [4,7].

On the other hand, for $\Delta m_{\text{atm}}^2 \lesssim 10^{-3} \text{ eV}^2$ the CHOOZ bounds are not operative and, in principle, sizable $\nu_\mu \leftrightarrow \nu_e$ oscillations can occur. Even though it seems difficult to reconcile SK data with large ν_e mixing and relatively small Δm_{atm}^2 [4,7], nevertheless, this possibility has been recently claimed in some phenomenological models, including: a) ‘‘Threefold

maximal mixing” of solar and atmospheric neutrinos [8,9]; b) Three-flavor fits to solar, atmospheric, and accelerator evidence for neutrino oscillations [10–14]. Such models with large ν_e mixing are representative of the two possible identifications for Δm_{atm}^2 , which is taken equal to m^2 in model (a) and to δm^2 in model (b).

Since both models (a) and (b) have received considerable and continued attention in the neutrino physics literature¹, it seems useful to assess clearly their phenomenological status through a quantitative study of the Super-Kamiokande observations. In this work, we perform such an analysis and show that model (a) is disfavored (although not yet excluded), while model (b) is definitely ruled out. We also try to trace the origin of different claims by other authors.

II. THREEFOLD MAXIMAL MIXING

Threefold maximal mixing is defined by a democratic mixing matrix: $|U_{\alpha i}^2| = 1/3$ ($\alpha = e, \mu, \tau$; $i = 1, 2, 3$), so that all oscillation channels are open. The parameters m^2 and δm^2 are assumed to drive atmospheric and solar neutrino oscillations, respectively, so that $\delta m^2 \ll m^2$ can be assumed. The phenomenological implications of such model have been intensively studied by Harrison, Perkins, and Scott (see [8,15] and references therein).

As far as atmospheric neutrinos are concerned, one can take $m^2 = \Delta m_{\text{atm}}^2$ and $\delta m^2 \simeq 0$ (one mass scale dominance [4]). Since the mixing matrix elements (squared) are fixed to the common value $1/3$, the only parameter probed by SK data is m^2 . In the following, we discuss how matter effects and fits to the SK and CHOOZ data constrain the value of m^2 for threefold maximal mixing, and distinguish this scenario from twofold maximal mixing.

A. Oscillations in matter

The three-flavor oscillation probabilities for the case of $\delta m^2 \ll m^2 = \Delta m_{\text{atm}}^2$ and generic mixing can be found, e.g., in Appendix C of Ref. [18]. The specific case of threefold maximal mixing, obtained by taking $s_{\psi}^2 = 1/2$ and $s_{\phi}^2 = 1/3$, leads to the well-known results for vacuum oscillations:

$$P_{ee}^{\text{vac}} = 1 - \frac{8}{9} S, \quad (8)$$

$$P_{\mu e}^{\text{vac}} = \frac{4}{9} S, \quad (9)$$

$$P_{\mu\mu}^{\text{vac}} = 1 - \frac{8}{9} S, \quad (10)$$

where $S = \sin^2(m^2 L/4E)$.

In the threefold maximal mixing scenario (as well as in any model with large ν_e mixing) atmospheric neutrino oscillations below the horizon are heavily affected by Earth matter

¹For pre-SK studies see, e.g., Ref. [15] for case (a), Ref. [16] for case (b), and [17] for a general review of three-flavor scenarios.

effects [19], as emphasized, e.g., in [4,9,18,20–24], and recently realized also in [8].² Since the majority of upgoing leptons in SK are produced by neutrinos traversing only the Earth mantle, the bulk of matter effects can be understood by assuming a constant electron density ($N_e \simeq 2 \text{ mol/cm}^3$) along the neutrino trajectory. In this case, the flavor oscillation probabilities $P_{\alpha\beta}^{\text{mat}}$ depend on the (constant) matter-induced squared mass term A :

$$A = 2\sqrt{2} G_F N_e E \simeq 0.3 \times 10^{-3} \frac{E}{\text{GeV}} [\text{eV}^2] , \quad (11)$$

and can be computed analytically. Explicit expressions for $P_{\alpha\beta}^{\text{mat}}$ can be found in [18]. Two limit cases are particularly illuminating for understanding matter effects in threefold maximal mixing: low energy events ($A \ll m^2$) and high energy events ($A \gg m^2$).

The regime $A \ll m^2$ corresponds (for $m^2 \sim 10^{-3} \text{ eV}^2$) to $E \ll 3 \text{ GeV}$, and is thus relevant for the so-called sub-GeV (SG) atmospheric neutrino events. In this limit it turns out that [18]

$$P_{ee}^{\text{mat}} = P_{ee}^{\text{vac}} , \quad (12)$$

$$P_{\mu e}^{\text{mat}} = P_{\mu e}^{\text{vac}} , \quad (13)$$

$$P_{\mu\mu}^{\text{mat}} = P_{\mu\mu}^{\text{vac}} - \delta P , \quad (14)$$

where the term δP , for threefold maximal mixing, is given by:

$$\delta P = \frac{1}{3} \sin^2 \left(\frac{AL}{6E} \right) \quad (15)$$

$$\simeq \frac{1}{3} \sin^2(L/R_\oplus) \quad (16)$$

$$\simeq \frac{1}{3} \sin^2(2 \cos \Theta) . \quad (17)$$

In the above equations, Θ is the neutrino zenith angle ($L \simeq 2R_\oplus \cos \Theta$), and the accidental equality $A/6E \simeq R_\oplus$ for $N_e = 2 \text{ mol/cm}^3$ has been used. It thus appears that, in the sub-GeV sample, no significant matter effects can be expected for electron events, while the muon event rate should be further suppressed via the energy-independent term $\delta P \lesssim 10\%$. This term is generated by the effective splitting in matter of the two vacuum-degenerate eigenstates (ν_1, ν_2) [18,20,24].

The regime $A \gg m^2$ corresponds (for $m^2 \sim 10^{-3} \text{ eV}^2$) to $E \gg 3 \text{ GeV}$, and is thus relevant for the so-called upward through-going muon events (UP μ). In this limit it turns out that

$$P_{ee}^{\text{mat}} = 1 , \quad (18)$$

$$P_{\mu e}^{\text{mat}} = 0 , \quad (19)$$

$$P_{\mu\mu}^{\text{mat}} = 1 - \sin^2 \left(\frac{2}{3} m^2 \frac{L}{4E} \right) , \quad (20)$$

²This contrasts with solar neutrino oscillations, where matter effects are ineffective for threefold maximal mixing [25].

namely, threefold maximal mixing in matter becomes equivalent to twofold maximal mixing in vacuum [9,23], but with an effective squared mass difference decreased by a factor 2/3: $m_{\text{mat}}^2 = 2/3 m^2$ [8,9].

The above approximations are useful for a qualitative understanding of oscillations in matter, but cannot be applied in the regime of intermediate energies [$E \sim O(3 \text{ GeV})$], typical of the so-called multi-GeV (MG) events. Therefore, for numerical calculations we prefer to solve exactly, at any energy, the neutrino propagation equations along the Earth density profile. Details of our computation technique can be found in our previous works [4,7,18,22]; see, in particular, [4] for SK observables. Representative results of matter effects for threefold maximal mixing are shown in Fig. 1.

Figure 1 shows the SK distributions of sub-GeV electrons and muons (SG e and SG μ), of multi-GeV electrons and muons (MG e and MG μ) and of upward-going muons (UP μ), as a function of the lepton zenith angle θ , for $m^2 = 10^{-3} \text{ eV}^2$. In each bin, the lepton rates are normalized to their expectations in the absence of oscillations. The SK data (dots with error bars) refer to 45 kTy [3]. The solid histograms (blue in color) represent the predictions of threefold maximal mixing including matter effects; the dashed histograms (red in color) are obtained by assuming pure vacuum oscillations. As expected from the previous discussion, matter effects are negligible for the SG e distribution, while they generate an additional suppression for the SG μ distribution. The UP μ distribution is less “tilted” in the presence of matter oscillations, as a result of the smaller effective squared mass at high energy: $m_{\text{mat}}^2 \simeq 2/3 m^2$. At “intermediate energies” (MG events), both the e and μ distributions are affected by oscillations in matter. It can be seen that matter effects help the fit to SG and MG data, but slightly worsen the fit to UP μ data, as compared to pure vacuum oscillations.

B. Data analysis

It has recently been claimed that, including matter effects, threefold maximal mixing with $m^2 \sim 10^{-3}$ represents a good fit to all the SK data [8,9,26]. Although matter effects certainly help to fit the data in such model, the statements in [8,9,26] seem too optimistic as compared with our detailed analysis of either 33 kTy [4] or 45 kTy [7] SK data, which disfavors scenarios with large values of $|U_{e3}^2|$, including threefold maximal mixing ($|U_{e3}^2| = 1/3$). In this section, we clarify and further corroborate our previous results, and argue about the differences with the more optimistic claims in [8,9,26].

Figure 2 shows the SK distributions for threefold maximal mixing ($m^2 = 10^{-3} \text{ eV}^2$, matter effects included), as compared with the case of twofold maximal mixing $\nu_\mu \leftrightarrow \nu_\tau$ (at $m^2 = 1$ and $3 \times 10^{-3} \text{ eV}^2$). The differences among 2ν and 3ν distributions are relatively small for SG e , SG μ , and MG e events, while they are more significant for MG μ and UP μ events. Let us thus focus on the latter two samples. For MG μ , threefold maximal mixing overestimates the bin rate around the horizon ($\cos\theta \in [-0.2, 0.2]$), as compared to twofold maximal mixing. This effect has two components: (i) the relatively low value of m^2 implied by CHOOZ for 3ν maximal mixing ($\lesssim 10^{-3} \text{ eV}^2$), with respect to the best-fit value for 2ν maximal mixing ($\sim 3 \times 10^{-3} \text{ eV}^2$); and (ii) the further “matter” suppression of m^2 by a factor 2/3 for the highest-energy part of the MG μ sample (partially contained events). Both components tend to increase the typical oscillation wavelength for 3ν maximal mixing (as

compared to the 2ν case), so that longer pathlengths are required to get effective muon suppression.³ Concerning $\text{UP}\mu$'s one faces similar problems. Threefold maximal mixing is unable to fit the slope suggested by the data pattern, both because m^2 is necessarily low, and because of the further effective $2/3$ suppression of m^2 in matter. For $\text{UP}\mu$ events, threefold maximal mixing with $m^2 = 10^{-3} \text{ eV}^2$ is phenomenologically equivalent to twofold maximal mixing with $m^2 = 0.67 \times 10^{-3} \text{ eV}^2$ —a value which is definitely below the 99% C.L. range allowed by $\text{UP}\mu$ data alone (see Fig. 9 in [3]). Summarizing, Fig. 2 shows that 2ν and 3ν maximal mixing are discriminated mainly by $\text{MG}\mu$ and $\text{UP}\mu$ data; the low value of m^2 allowed by CHOOZ for 3ν maximal mixing ($\lesssim 10^{-3} \text{ eV}^2$) prevents a good fit to the $\text{MG}\mu$ rate around the horizon and to the slope of $\text{UP}\mu$'s, in contrast to the very good fit provided with 2ν maximal mixing at $m^2 \sim 10^{-3} \text{ eV}^2$.

Figure 3 shows χ^2 -fits to all the SK data (SG+MG+UP) for twofold and threefold maximal mixing, as a function of m^2 . Details of our statistical analysis can be found in [4]. The dot-dashed line (black in color) refers to the twofold maximal mixing, characterized by $\chi^2_{\min} \simeq 20$ and $m^2 \simeq 2.8 \times 10^{-3} \text{ eV}^2$ at best fit. The allowed range of m^2 is in very good agreement with the SK official analysis (compare with Fig. 10 in [3]). The dashed line (red in color) refers to threefold maximal mixing (SK data only), which in principle could provide a good fit for $m^2 \simeq 4 \times 10^{-3} \text{ eV}^2$, if such value were not excluded by CHOOZ. Adding the CHOOZ constraint (solid line, blue in color), the best fit is pushed to lower m^2 values and the value of χ^2_{\min} increases up to ~ 35 . Although this is not high enough to be ruled out, it is definitely worse than in the two-flavor case since, as observed in Fig. 2, 3ν maximal mixing fails to fit the $\text{UP}\mu$ and horizontal $\text{MG}\mu$ data. The situation would be even worse if matter effects were hypothetically switched off (dotted line, green in color).

The quantitative results of Fig. 3 indicate that the two-flavor and three-flavor scenarios with maximal mixing differ by $\Delta\chi^2 \sim 15$; therefore, if 3ν mixing is left unconstrained, the minimum will fall close to the 2ν case with zero ν_e mixing, while 3ν cases with large ν_e mixing will be disfavored. This is indeed the pattern found in the 3ν parameter space analysis of [4,7]. However, it should also be said that the 3ν maximal mixing fit to the SK data is not terribly bad, so it is wiser to wait for higher statistic in the $\text{MG}\mu$ and $\text{UP}\mu$ samples, or for more stringent reactor bounds on m^2 , before (dis)proving it definitely.

We conclude by examining possible reasons for the more optimistic results found in [8,9,26]. The work [26] did not include $\text{UP}\mu$ data, which contribute to worsen the fit for threefold maximal mixing. The work [9], moreover, considered only the up-down asymmetry for SG and MG data, thus excluding also the important information given by nearly horizontal $\text{MG}\mu$. The comparison with [8] is more delicate since, in principle, a data set similar to ours is analyzed. However, the lack of details and of explicit comparison with SK calculations in [8] prevents us to trace the source of the differences. Nevertheless, a comparison of our Fig. 3 with the corresponding results in [8] (see their Fig. 7) gives us some hints. It is quite evident that, for the two-flavor oscillation case, the preferred range for m^2

³The recent ten-bin $\text{MG}\mu$ data from SK rapidly drop at the horizon, and show no evidence (within errors) for a wavelength significantly longer than for the pure 2ν best-fit case at $m^2 \sim 3 \times 10^{-3} \text{ eV}^2$; see Fig. 3 in [3].

in [8] is biased towards low values, as compared with our range in Fig. 3 and with the SK official analysis (Fig. 10 in [3]). This is surprising, since the authors of [8] include also the old Kamiokande data, which should rather pull the fit towards slightly higher m^2 values. A bias towards low m^2 's artificially increases the chance that threefold maximal mixing can survive below CHOOZ bounds.

In conclusion, we find that threefold maximal mixing is significantly disfavored with respect to twofold maximal mixing ($\Delta\chi^2_{\min} \simeq 15$). The most discriminating data samples are MG μ and UP μ . However, further reactor data, and atmospheric neutrino data in the intermediate and high energy range, are needed in order to (dis)prove the model with higher confidence. The Super-Kamiokande collaboration itself has now all the tools to investigate quantitatively generic three-flavor scenarios including matter effects [27], so as to settle definitely some differences existing among present independent phenomenological analyses.

III. SOLAR + ATMOSPHERIC + LSND SCENARIOS

The sources of evidence for neutrino oscillations coming from solar and atmospheric neutrinos, as well as from the Liquid Scintillator Neutrino Detector experiment (LSND) [28] suggest three widely different neutrino mass square differences. Therefore, they cannot be reconciled in a single three-flavor scenario, unless some data are “sacrificed” [17]. One possibility [16], indicated as “case 3c” in Table VI of Ref. [17], has been recently revived in several works [10–14] which claim that it can provide a good fit to the SK atmospheric neutrino data, and also explain most of the solar neutrino deficit and the LSND data. In this section, we briefly discuss such scenario, and show quantitatively that it is ruled out by the SK atmospheric ν data.

The starting hypothesis is that $(\delta m^2, m^2) = (\Delta m^2_{\text{atm}}, \Delta m^2_{\text{LSND}})$, so that: (i) concerning solar neutrinos, both δm^2 and m^2 drive energy-averaged oscillations, which can explain the bulk of the solar neutrino deficit but not possible distortions in the energy spectrum; (ii) for atmospheric neutrinos, δm^2 drives the energy-dependent oscillations while m^2 gives an energy-averaged contribution; and (iii) for LSND, δm^2 can be taken effectively equal to zero, and oscillations are driven by $m^2 \sim O(1 \text{ eV}^2)$. In order to get the desired $P_{\mu e}^{\text{LSND}} = 4U_{e3}^2 U_{\mu 3}^2 \sin^2(m^2 L/4E) \sim \text{few per mill}$, the heavy state ν_3 is taken close to ν_τ ($U_{\tau 3}^2 \sim 1$), so that (ν_1, ν_2) are basically linear combinations of (ν_μ, ν_e) . By choosing nearly maximal mixing between $\nu_{1,2}$ and $\nu_{e,\mu}$, one then hopes to solve both the atmospheric neutrino anomaly and the solar neutrino deficit through dominant $\nu_\mu \leftrightarrow \nu_e$ oscillations driven by $\delta m^2 = \Delta m^2_{\text{atm}}$. The CHOOZ results then constrain δm^2 to range below $\sim 10^{-3} \text{ eV}^2$.

The mixing matrix for such scenario is rather strongly constrained by the requirement to fit solar, atmospheric, and LSND data. Indeed, the phenomenological matrices found in different works [10–14] are similar to each other, with small differences which are not important for our discussion. For definiteness, we report the matrix $U_{\alpha i}$ found in the paper [10] by Thun and McKee:

$$U_{\alpha i} \simeq \begin{pmatrix} 0.78 & 0.60 & 0.18 \\ -0.61 & 0.66 & 0.44 \\ 0.15 & -0.45 & 0.88 \end{pmatrix}. \quad (21)$$

In the same work [10], the best-fit value of m^2 is 0.4 eV^2 . Such choice for the mass-mixing parameters will be dubbed “the Thun-McKee scenario.”

Figure 4 shows the expected zenith distributions of lepton events in SK for the Thun-McKee scenario at $\delta m^2 = 10^{-3} \text{ eV}^2$, with and without matter effects. Matter effects play an important role in suppressing oscillations: when the (ν_μ, ν_e) mixing is (nearly) maximal in vacuum, it can only be smaller in matter. A similar pattern, in fact, is found for pure two-flavor $\nu_\mu \leftrightarrow \nu_e$ oscillations (see Fig. 9 of [4]). It can be seen in Fig. 4 that the Thun-McKee scenario is ruled out by the SK data for several reasons: (i) the SGe and MGe upgoing event rates are overestimated; (ii) the SG μ and MG μ up-down asymmetries are largely underestimated; (iii) the predicted UP μ rate is too low; and (iv) matter effects play an important role in worsening the fit.⁴ The situation is not improved by lowering the value of m^2 , as it can be seen in Fig. 5. We find that the Thun-McKee scenario, in any case, gives $\chi^2 \gtrsim 100$, and is thus definitely ruled out by the SK data *alone*.⁵

In addition, the Thun-McKee scenario does not provide a good fit to the current reactor and accelerator oscillation data, including LSND, as already observed in [17]. In fact, the quantitative three-flavor analysis of laboratory data performed in [29] selects only two possible solutions at 90% C.L., namely, $\nu_3 \sim \nu_e$ or $\nu_3 \sim \nu_\mu$, while $\nu_3 \sim \nu_\tau$ is highly disfavored.

In conclusion, we find that the Thun-McKee scenario is not a viable explanation of the solar+atmospheric+LSND data. The SK data are sufficient to rule it out definitely. The much more optimistic claims in [10–14] are not substantiated by quantitative calculations of electron and muon event rates in SK.

IV. CONCLUSIONS

We have made a detailed comparison of the SK atmospheric ν data with the predictions of two popular models characterized by large $\nu_\mu \leftrightarrow \nu_e$ transitions: (a) Threefold maximal mixing [8,9]; b) Three-flavor fits to solar, atmospheric, and LSND data [10–14]. We have found that model (a) [8] is disfavored (although not yet excluded), while model (b) [10–14] is definitely ruled out. The origin of different claims by other authors has been elucidated.

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⁴Notice that the works [10–14] do not include matter effects, adopt approximate estimates of the SK event rates, and basically use only a subset of the data, namely, the μ/e ratio of contained events.

⁵The situation is similar for the slightly different mass-mixing parameter choices performed in [11–14].

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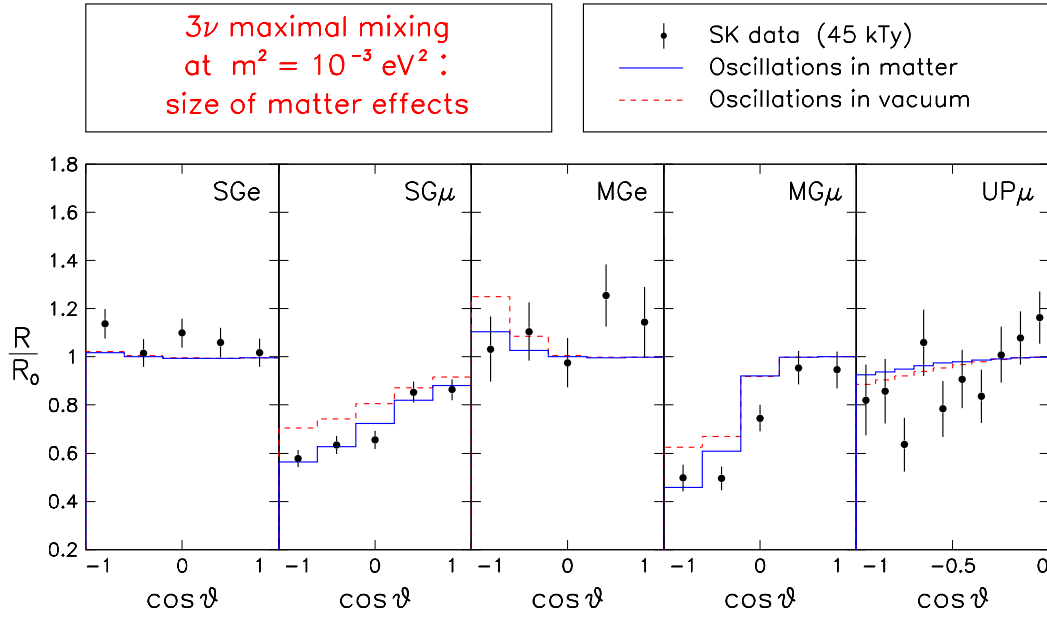


Fig. 1. Threefold maximal mixing scenario at $m^2 = 10^{-3} \text{ eV}^2$. From left to right: zenith distributions of sub-GeV (SG) electrons (e) and muons (μ), of multi-GeV (MG) e and μ , and of upward going muons (UP μ), normalized in each bin to the standard expectations (no oscillation). Solid line: predictions including matter effects. Dashed lines: predictions without matter effects. Dots with error bars: Super-Kamiokande data.

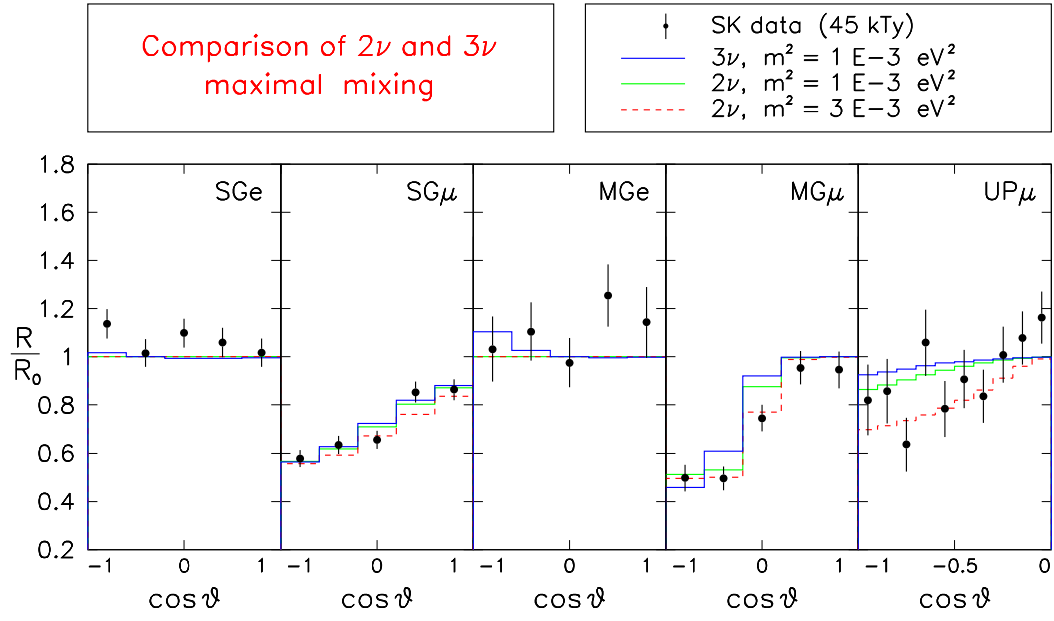


Fig. 2. Comparison of twofold (2ν) and threefold (3ν) maximal mixing predictions.

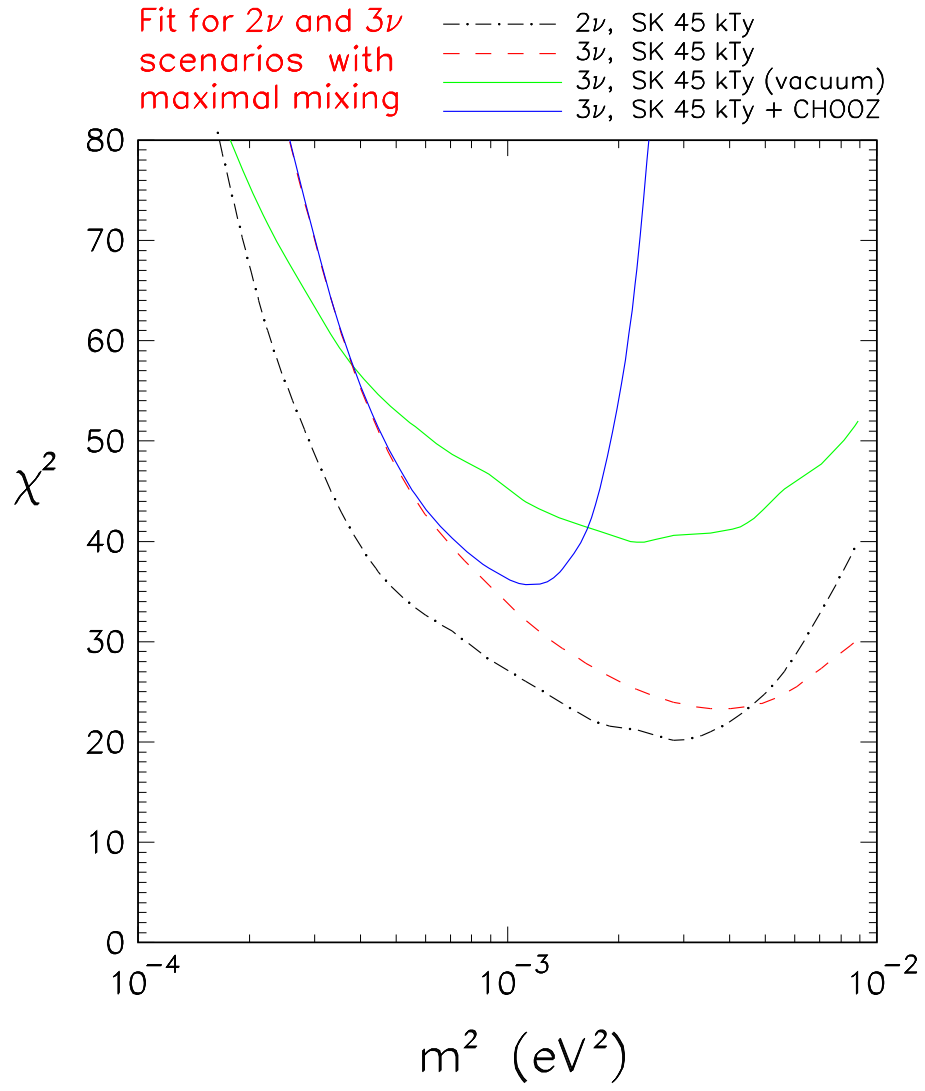


Fig. 3. Fit to twofold and threefold maximal mixing scenarios for variable m^2 .

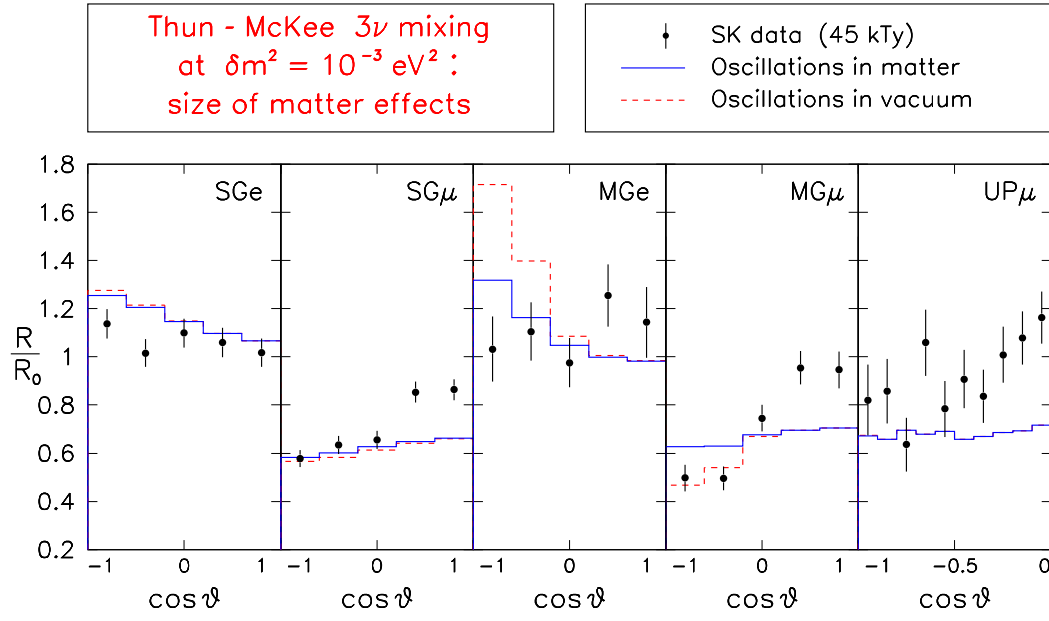


Fig. 4. Thun-McKee 3ν mixing scenario [10] at $\delta m^2 = 10^{-3} \text{ eV}^2$, with and without matter effects.

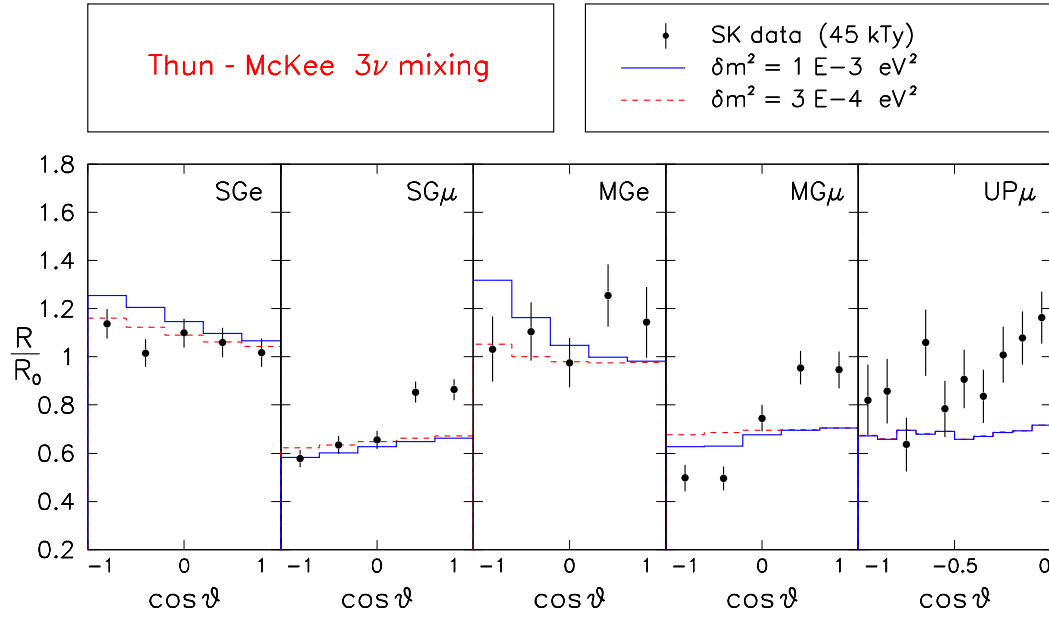


Fig. 5. Thun-McKee 3ν mixing scenarios at $\delta m^2 = 10^{-3}$ and $3 \times 10^{-4} \text{ eV}^2$ (matter effects included).